Faraday Rotation

Physics 3600 - Advanced Physics Lab - Summer 2021

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I. INTRODUCTION

More than 150 years ago, using very crude equipment, Michael Faraday discovered that *linearly polarized* light traveling through a substance experiences a rotation when a magnetic field is applied to the material. It was the first observation that a magnetic field can affect light.

The amount of rotation of the polarization (ϕ) was found to be proportional to the magnitude of the magnetic field (**B**) and to the length of the sample (*L*),

where the constant of proportionality (V) is called the Verdet constant.

A linearly polarized beam of light, one that has a unique electric field *E*-vector direction, can be decomposed into two circularly polarized beams of equal intensity. These right- and left-circularly polarized beams propagate simultaneously, but are independent of each other (orthogonal). An applied B-field causes the material to become *optically active*, producing different refractive indices for the two beams, so that they travel at different phase velocities. Thus, when the two beams exit the material they have a different phase relationship, which is manifest in a rotation of the *E*-vector of the combined beams.

In this experiment you will set up an apparatus for measuring very small rotations (< 10^{-3} deg) of polarized light passing though samples placed in a magnet solenoid. The Verdet constant will be determined for glass, water and plastic. (See the example lab writeup, <u>Faraday Rotation Webarticle</u>, but do not use their values for the Verdet constants. Instead, do your own search on the web and reference them.)

APPARATUS (be sure to write down model numbers in your report) optical breadboard, two Glan (calcite) polarizers with rotating mounts, lens, collimated laser diode, two silicon photodiodes with 10 KΩ resistors, frequency generator, SRS lock-in amplifier, oscilloscope, electromagnet/power supply, 20 mm cuvette cells with 1.1 mm thick BK-7 glass (~SiO₂) windows, plastic disk

III. PROCEDURE

A. Calibrate Magnetic Field

Connect a power supply to electromagnet solenoid *through* a current meter, but **do not exceed 3 amps**. With the Hall-probe gaussmeter, measure the B-field in the magnet center at a few current values (I).

 $\Box \quad \underline{Plot B(I) and fit to get the slope coefficient \alpha, where B = \alpha I.}$

B. Initial Setup

Make sure the GW frequency generator is <u>OFF</u> before connecting/disconnecting the laser diode.

Connect the TTL/CMOS output of the GW frequency generator to the laser diode.

Connect the right-hand generator output to the lock-in amplifier reference (REF).

Pull out the CMOS knob and rotate to minimum (full CCW).

Turn <u>**ON**</u> generator, set to $f \sim 500$ Hz, and increase CMOS adjustment to maximum (CW) while looking for red spot on a card.

You are now ready to align the optical components.

C. Alignment of Optics



Using a white card, make sure that the laser beam passes through the holes in the magnet pole pieces. Adjust the photodiode PD-1 so that the beam hits its center.

Make sure that the beam passes through the center of polarizers POL-1 and POL-2.

Make sure that the polarizer POL-2 is rotated so that the *side exit beam* is parallel to the table. Adjust the photodiode PD-2 so that the beam hits its center.

Use the Polaroid sheet polarizer to check that the beam exiting POL-1 is linearly polarized at approximately 45 deg away from vertical.

Use the Polaroid sheet to determine the polarization states of the two beams exiting POL-2.

□ What are the specific polarization states of the two beams exiting POL-2?

Connect the two BNC cables from the photodiodes into the scope Chnl-1 and Chnl-2 inputs.

Describe the waveforms from the photodiodes.

Loosen the small screw that is on the lower-back of the polarizer POL-1 by 1/4 turn. Rotate the angle on POL-1 to get equal photodiode voltages on the scope. Lightly retighten the small screw.

D. Configure Lock-in Amplifier

Now, switch the two BNC PD outputs to the *A* and *B* differential inputs of the lock-in amplifier.

Use the frequency generator OUTPUT as the lock-in reference input.

Set the time constant to 300 ms.

Configure the lock-in amplifier to display **R** and **0**.

 $R=|V_1-V_2|$ and θ is the phase between R and the reference input. See Appendix-II for lock-in information.

E. Calibrate Rotation (see Appendix-I)

The FR angle is given by

 $\phi = \beta \rho ,$

where β is a constant determined by the sensitivity of the apparatus, and ρ is the "polarization."

For small rotations, $\phi \ll 1$ radian, the polarization ρ is given simply by the voltage difference of the two PDs divided by their sum,

 $\rho = (V_1 - V_2) / (V_1 + V_2).$

Here you will measure β for the optical setup. The constant β is determined from $\beta = d\phi/d\rho$ by varying ϕ via rotating the polarizer POL-1 in very small increments and measuring ρ .

Now, select the *A*-minus-*B* function on the lock-in amplifier, which makes $R = |V_1 - V_2|$. Rotate the micrometer on POL-1 to minimize *R*.

Note: *R* is a magnitude, so when V_1-V_2 changes sign, the angle θ changes by ~180 deg. Select A-input **only** function, record *R* as V_1 for one of the PDs. Exchange the A and B input BNC cables and record *R* as V_2 for the other PD.

Now, set up the *A*-minus-*B* function again on the lock-in amplifier, which makes $R = |V_1-V_2|$.

Make a table containing the columns: D, R, θ , and $\rho = R / (V_1 + V_2)$.

Next, adjust the micrometer in small steps ($\Delta D=0.001$ ", see *Appendix-I*) to 3 or 4 positions on both sides of the minimum R value and compute $\rho(D)$. You do not need to measure V₁ and V₂ separately as a function of D.

Note, to eliminate errors from the mechanical backlash, always set the micrometer position D by rotating in the **same direction** as you go through the micrometer position where R is a minimum.

- \Box <u>Plot and curve fit $\rho(D)$.</u>
- $\Box \quad \underline{\text{Compute } \beta = d\phi/dp \text{ from the slope } dp/dD \text{ and } d\phi/dD \text{ (see Appendix-I).}}$
- Convert the noise level (*R*-value fluctuations) of the lock-in into units of μrad?

F. Measure Faraday Rotation

Make a table with columns: I, R, θ , and $\rho = R / (V_1 + V_2)$.

Water + Glass Cell

Place the water-filled cell between the magnet pole pieces. Adjust the POL-1 micrometer to minimize *R*.

Select A-input function, record R as V₁. Exchange the A and B input BNC cables, record R as V₂.

 \Box <u>Did (V₁+V₂) change from the value without the cell?</u> <u>Explain.</u>

Again, set up the **A-minus-B function** on the lock-in amplifier, which makes $R = |V_1-V_2|$.

Record *R* and θ from the lock-in readings while varying the magnet current I for positive and negative I-values up to I = ±3.0 A.

 $\label{eq:product} \Box \quad \underline{\mathsf{Plot}} \; \rho_{\mathsf{WG}}(I). \quad \underline{\mathsf{Curvefit}} \; \rho_{\mathsf{WG}}(I) \; to \; obtain \; d\rho_{\mathsf{WG}}/dI.$

Glass Cell Only

Place the empty cell between the magnet pole pieces. Adjust the POL-1 micrometer to minimize *R*.

Again, select A-input function, record R as V_1 . Exchange the A and B input BNC cables, record R as V_2 .

 \Box <u>Did V₁+V₂ change from the value with the water-filled cell?</u> <u>Explain.</u>

Now, set up the *A*-minus-*B* function again on the lock-in amplifier, which makes $R = |V_1-V_2|$. Record *R* and θ from the lock-in amplifier while varying the magnet current I for positive and negative I-values up to I = ±3.0 A.

 $\Box \quad \underline{\text{Plot}} \ \rho_{\text{WG}}(I \ \underline{)}. \quad \underline{\text{Curvefit}} \ \rho_{\text{WG}}(I) \ \underline{\text{to obtain}} \ d\rho_{\text{WG}}/dI.$

<u>Plastic</u>

□ <u>Collect data for measuring the Verdet constant of plastic.</u>

Verdet Constants

□ <u>Compute the Verdet constant and uncertainty for BK-7 glass from</u>

$$V_G = \frac{\beta}{\alpha} \frac{1}{L_G} \frac{d\rho_G}{dI}.$$

□ <u>Compute the Verdet constant and uncertainty for water only from</u>

$$V_W = \frac{\beta}{\alpha} \frac{1}{L_W} \left[\frac{d\rho_{WG}}{dI} - \frac{d\rho_G}{dI} \right].$$

Rapid Data Collection

Collect some data by sweeping the field by hand. Use three analog-to-digital (ADC) channels on the **Labjack U6**. Input the voltage from the 0.2 Ω shunt resistor to measure the current into channel 0-1; input the lock-in R output into channel 2-3; and input the θ output into channel 4-5.

Reduce the lock-in time constant to 10 ms, which will allow you to sweep the magnet power supply from 0 to ~3 A in about 10 seconds. Start the program LJLogUD and click on **write to file** before you start sweeping. Sweep the current up from 0 to 3 A, then down 3 A to 0. Switch the current polarity from positive to negative, followed by sweep up then down.

Plot the voltages versus current and reverse the sign of the R voltage for the negative current direction. Make sure that the data up/down follows along a single line without showing an open loop. An open loop indicates that either the time constant is too long or that the current was swept too fast. Convert the plot to $\phi(B)$.

G. Summary

- □ Compare your values of the Verdet constant to expected values, in units of µrad/Gcm. Find accepted values from your own web search.
- □ Put these values in a table in your report and discuss.
- □ Discuss the origin and wavelength dependence of the Verdet constant.

IV. Appendix: Calibrating Faraday Rotation Angle

Here are notes for calibrating the Faraday rotation setup. They are used to determine the coefficient $\beta = d\phi/d\rho$, which relates the FR angle ϕ to the measured polarization ρ .

First, note that the angle of the linear polarizer POL-1 is rotated by the micrometer. The angle of the polarizer

 ϕ is a function of the micrometer setting D. The calcite polarizer is installed in a rotating mount that has two modes of rotation, gross and fine. Do the gross adjustment only if the polarization is not approximately set to pass polarization at 45 deg from the vertical. Gross rotation is achieved by unscrewing the small locking screw located in the lower rear about 1/4 turn, then rotating the whole barrel. Fine rotation is accomplished by lightly tightening the locking screw then turning the micrometer on the top. Notice that each mark on the turning barrel of the micrometer corresponds to $\Delta D=0.001$ " movement of the micrometer plunger. Also notice that the micrometer plunger is r=1.5" from the center of the polarizer. D and r are related by the formula $\Delta D = r \Delta \phi$, where D and r are in the same units and ϕ has units of radians (57.3 deg/rad).



For calibration, the polarization $\rho(D)$ is found by measuring ρ for several values of D differing by $\Delta D=0.001$ ", on either side of $\rho=0$. Make sure to always set the micrometer position by rotating in the same direction to avoid backlash. Then, $d\rho/dD$ is determined from the slope of the $\rho(D)$ plot. Rotating the micrometer by one mark ($\Delta D=0.001$ ") corresponds to a rotation of $\phi=667$ microradians, so that

$$\frac{d\phi}{dD} = 6.7 \times 10^5 \,\frac{\mu rad}{in}$$

Finally, the coefficient β , in units of μ rad, is computed from the measured dp/dD using

$$\beta = \frac{d\phi}{dD} \times \frac{1}{d\rho/dD}.$$

WHAT IS A LOCK-IN AMPLIFIER?

Lock-in amplifiers are used to detect and measure very small AC signals - all the way down to a few nanovolts! Accurate measurements may be made even when the small signal is obscured by noise sources many thousands of times larger.

Lock-in amplifiers use a technique known as phase-sensitive detection to single out the component of the signal at a specific reference frequency AND phase. Noise signals at frequencies other than the reference frequency are rejected and do not affect the measurement.

Why use a lock-in?

Let's consider an example. Suppose the signal is a 10 nV sine wave at 10 kHz. Clearly some amplification is required. A good low noise amplifier will have about 5 nV/ \sqrt{Hz} of input noise. If the amplifier bandwidth is 100 kHz and the gain is 1000, then we can expect our output to be 10 μ V of signal (10 nV x 1000) and 1.6 mV of broadband noise (5 nV/ \sqrt{Hz} x $\sqrt{100}$ kHz x 1000). We won't have much luck measuring the output signal unless we single out the frequency of interest.

If we follow the amplifier with a band pass filter with a Q=100 (a VERY good filter) centered at 10 kHz, any signal in a 100 Hz bandwidth will be detected (10 kHz/Q). The noise in the filter pass band will be 50 μ V (5 nV/ $\sqrt{Hz} \times \sqrt{100}$ Hz $\times 1000$) and the signal will still be 10 μ V. The output noise is much greater than the signal and an accurate measurement can not be made. Further gain will not help the signal to noise problem.

Now try following the amplifier with a phasesensitive detector (PSD). The PSD can detect the signal at 10 kHz with a bandwidth as narrow as 0.01 Hz! In this case, the noise in the detection bandwidth will be only 0.5 μ V (5 nV/ $\sqrt{Hz} \times \sqrt{.01}$ Hz x 1000) while the signal is still 10 μ V. The signal to noise ratio is now 20 and an accurate measurement of the signal is possible.

What is phase-sensitive detection?

Lock-in measurements require a frequency reference. Typically an experiment is excited at a fixed frequency (from an oscillator or function generator) and the lock-in detects the response from the experiment at the reference frequency. In the diagram below, the reference signal is a square wave at frequency ω_{r} . This might be the sync output from a function generator. If the sine output from the function generator is used to excite the experiment, the response might be the signal waveform shown below. The signal is $V_{sig}sin(\omega_{r}t + \theta_{sig})$ where V_{sig} is the signal amplitude.

The SR830 generates its own sine wave, shown as the lock-in reference below. The lock-in reference is $V_L sin(\omega_L t + \theta_{ref})$.



SR830 amplifies the signal and the

The SR830 amplifies the signal and then multiplies it by the lock-in reference using a phase-sensitive detector or multiplier. The output of the PSD is simply the product of two sine waves.

$$V_{psd} = V_{sig}V_{L}sin(\omega_r t + \theta_{sig})sin(\omega_L t + \theta_{ref})$$

 $= \frac{1/2 V_{sig} V_{L} cos([\omega_{r} - \omega_{L}]t + \theta_{sig} - \theta_{ref})}{1/2 V_{sig} V_{L} cos([\omega_{r} + \omega_{L}]t + \theta_{sig} + \theta_{ref})}$

The PSD output is two AC signals, one at the difference frequency $(\omega_r - \omega_L)$ and the other at the sum frequency $(\omega_r + \omega_L)$.

If the PSD output is passed through a low pass filter, the AC signals are removed. What will be left? In the general case, nothing. However, if ω_r equals ω_L , the difference frequency component will be a DC signal. In this case, the filtered PSD output will be

$$V_{psd} = 1/2 V_{sig} V_{L} \cos(\theta_{sig} - \theta_{ref})$$

This is a very nice signal - it is a DC signal proportional to the signal amplitude.

Narrow band detection

Now suppose the input is made up of signal plus noise. The PSD and low pass filter only detect signals whose frequencies are very close to the lockin reference frequency. Noise signals at frequencies far from the reference are attenuated at the PSD output by the low pass filter (neither wnoise- ω_{ref} nor $\omega_{noise} + \omega_{ref}$ are close to DC). Noise at frequencies very close to the reference frequency will result in very low frequency AC outputs from the PSD ($|\omega_{noise} - \omega_{ref}|$ is small). Their attenuation depends upon the low pass filter bandwidth and roll-off. A narrower bandwidth will remove noise sources very close to the reference frequency, a wider bandwidth allows these signals to pass. The low pass filter bandwidth determines the bandwidth of detection. Only the signal at the reference frequency will result in a true DC output and be unaffected by the low pass filter. This is the signal we want to measure.

Where does the lock-in reference come from?

We need to make the lock-in reference the same as the signal frequency, i.e. $\omega_r = \omega_L$. Not only do the frequencies have to be the same, the phase between the signals can not change with time, otherwise $\cos(\theta_{sig} - \theta_{ref})$ will change and V_{psd} will not be a DC signal. In other words, the lock-in reference needs to be phase-locked to the signal reference.

Lock-in amplifiers use a phase-locked-loop (PLL) to generate the reference signal. An external reference signal (in this case, the reference square wave) is provided to the lock-in. The PLL in the lock-in locks the internal reference oscillator to this external reference, resulting in a reference sine wave at ω_r with a fixed phase shift of θ_{ref} . Since the PLL actively tracks the external reference, changes in the external reference frequency do not affect the measurement.

All lock-in measurements require a reference signal.

In this case, the reference is provided by the excitation source (the function generator). This is called an external reference source. In many situations, the SR830's internal oscillator may be used instead. The internal oscillator is just like a function generator (with variable sine output and a TTL sync) which is always phase-locked to the reference oscillator.

Magnitude and phase

Remember that the PSD output is proportional to $V_{sig}cos\theta$ where $\theta = (\theta_{sig} - \theta_{ref})$. θ is the phase difference between the signal and the lock-in reference oscillator. By adjusting θ_{ref} we can make θ equal to zero, in which case we can measure V_{sig} ($cos\theta=1$). Conversely, if θ is 90°, there will be no output at all. A lock-in with a single PSD is called a single-phase lock-in and its output is $V_{sig}cos\theta$.

This phase dependency can be eliminated by adding a second PSD. If the second PSD multiplies the signal with the reference oscillator shifted by 90°, i.e. $V_L sin(\omega_L t + \theta_{ref} + 90^\circ)$, its low pass filtered output will be

$$V_{psd2} = 1/2 V_{sig} V_{Lsin}(\theta_{sig} - \theta_{ref})$$

Now we have two outputs, one proportional to $\cos\theta$ and the other proportional to $\sin\theta$. If we call the first output X and the second Y,

$$X = V_{sig} cos \theta$$
 $Y = V_{sig} sin \theta$

these two quantities represent the signal as a vector relative to the lock-in reference oscillator. X is called the 'in-phase' component and Y the 'quadrature' component. This is because when θ =0, X measures the signal while Y is zero.

By computing the magnitude (R) of the signal vector, the phase dependency is removed.

$$R = (X^2 + Y^2)^{1/2} = V_{sig}$$

R measures the signal amplitude and does not depend upon the phase between the signal and lock-in reference.

A dual-phase lock-in, such as the SR830, has two PSD's, with reference oscillators 90° apart, and can measure X, Y and R directly. In addition, the phase θ between the signal and lock-in reference, can be measured according to

 $\theta = \tan^{-1} (Y/X)$

WHAT DOES A LOCK-IN MEASURE?

So what exactly does the SR830 measure? Fourier's theorem basically states that any input signal can be represented as the sum of many, many sine waves of differing amplitudes, frequencies and phases. This is generally considered as representing the signal in the "frequency domain". Normal oscilloscopes display the signal in the "time domain". Except in the case of clean sine waves, the time domain representation does not convey very much information about the various frequencies which make up the signal.

What does the SR830 measure?

The SR830 multiplies the signal by a pure sine wave at the reference frequency. All components of the input signal are multiplied by the reference simultaneously. Mathematically speaking, sine waves of differing frequencies are orthogonal, i.e. the average of the product of two sine waves is zero unless the frequencies are EXACTLY the same. In the SR830, the product of this multiplication yields a DC output signal proportional to the component of the signal whose frequency is exactly locked to the reference frequency. The low pass filter which follows the multiplier provides the averaging which removes the products of the reference with components at all other frequencies.

The SR830, because it multiplies the signal with a pure sine wave, measures the single Fourier (sine) component of the signal at the reference frequency. Let's take a look at an example. Suppose the input signal is a simple square wave at frequency f. The square wave is actually composed of many sine waves at multiples of f with carefully related amplitudes and phases. A 2V pk-pk square wave can be expressed as

 $S(t) = 1.273 sin(\omega t) + 0.4244 sin(3\omega t) + 0.2546 sin(5\omega t) + ...$

where $\omega = 2\pi f$. The SR830, locked to f will single out the first component. The measured signal will be 1.273sin(ωt), not the 2V pk-pk that you'd measure on a scope.

In the general case, the input consists of signal plus noise. Noise is represented as varying signals at all frequencies. The ideal lock-in only responds to noise at the reference frequency. Noise at other frequencies is removed by the low pass filter following the multiplier. This "bandwidth narrowing" is the primary advantage that a lock-in amplifier provides. Only inputs at frequencies at the reference frequency result in an output.

RMS or Peak?

Lock-in amplifiers as a general rule display the input signal in Volts RMS. When the SR830 displays a magnitude of 1V (rms), the component of the input signal at the reference frequency is a sine wave with an amplitude of 1 Vrms or 2.8 V pk-pk.

Thus, in the previous example with a 2 V pk-pk square wave input, the SR830 would detect the first sine component, 1.273sin(ω t). The measured and displayed magnitude would be 0.90 V (rms) (1/ $\sqrt{2}$ x 1.273).

Degrees or Radians?

In this discussion, frequencies have been referred to as f (Hz) and ω (2 π f radians/sec). This is because people measure frequencies in cycles per second and math works best in radians. For purposes of measurement, frequencies as measured in a lock-in amplifier are in Hz. The equations used to explain the actual calculations are sometimes written using ω to simplify the expressions.

Phase is always reported in degrees. Once again, this is more by custom than by choice. Equations written as $sin(\omega t + \theta)$ are written as if θ is in radians mostly for simplicity. Lock-in amplifiers always manipulate and measure phase in degrees.