**Coupled Electrical Oscillators**

[Physics 3600 - Advanced Physics Lab - Summer 2021](http://northeastern.edu/heiman/3600/index.html)

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**I. INTRODUCTION**

The objectives of this experiment are:

(1) explore the properties of a single LRC electrical oscillator circuit, including damping;

(2) study what happens when two oscillators are coupled and allowed to exchange energy.

More information on coupled oscillators can be found in the Appendix.

**II. APPARATUS**

small box with circuit, switches, connectors, two yellow (or large) inductor coils

capacitors - fixed standard and adjustable

storage scope with interfaced computer

curve fitting software (EastPlot , MatLab, Python)

**III. PROCEDURE**

***A. Measure Inductance***

Install the standard capacitor and one inductor in the A-circuit. Connect the +5 VDC source to the box. Connect the A-output to Chnl-1 of the scope. Connect the Trigger output to the scope trigger. Look for the distorted square wave output having oscillations at the edges of the pulse. Find the decaying oscillation of the A-circuit that occurs when the square wave pulse turns **off**. Note that the square wave pulse turning on or off is similar to a hammer striking a bell to start it ringing (oscillating).

1. Measure the capacitance of the standard capacitor using the Extech Digital MultiMeter (DMM).

2. View the oscillation where the excitation pulse goes to V=0, which has the largest number of cycles.

3. Measure only **one** oscillation period of each inductor in the A-circuit.

 a. Compute the two inductances and estimate their uncertainties.

 b. Compare the values of the inductors.

Using the instructions below, curve fit the oscillations to precisely measure the frequency and damping.

4. Store the *decaying* waveform of one inductor in the A-circuit.

First, transform the waveform to make the average amplitude equal to zero (σ in EasyPlot).

Curve-fit one or two periods to y=a\*sin(bx+c) to obtain values for the frequency b and phase c.

Then, knowing initial values for b and c, fit the oscillations to y=a\*sin(bx+c)exp(-dx/2).

 a. Plot the V(t) data along with the curve fit.

 b. What is the value of the damping coefficient γ (=d)?

 c. What is the value of the time constant of the damping, τ = 2/d?

 d. Compute precise values for L, R, Q, and uncertainties for the A-circuit? (see appendix)

 e. Compare the inductance uncertainty from the curve fit to that from the single period measurement.

**You must show the TA your plots at the end of each section before proceeding to the next section.**

***B. Compare Resistances***

 a. Compare the R computed from γ to the R of the coil measured with an ohmmeter. Discuss.

 b. According to theory, by what percentage does damping affect the frequency?

***C. Independent Oscillators***

Add the inductor and adjustable capacitor to the B-circuit and connect the output to Chnl-2 of the scope. With the coupling capacitor switched ***off*** (No) and "normal" excitation, adjust the B‑capacitor until both waveforms have the same period.

 a. Measure the adjustable B-capacitor with the DMM and compare to the standard capacitor.

***D. Unilateral Excitation of Coupled Oscillators***

With the coupling capacitor ***on*** (Yes), switch off the B‑circuit excitation (middle position of switch).

Again, use the oscillations that occur when the exciting pulse goes to zero volts.

Note that both the A and B waveforms periodically grow and decay in time.

Now, finely adjust the adjustable B-capacitor to achieve the best minima of one waveform.

1. Store waveforms of the A- and B-circuits.

2. Plot waveforms for the A- and B-circuits and their sum on one graph (shifted so they don’t overlap).

 a. Relate this electrical exchange to that in the mass/spring system in the lab room. Lower the mass, let go of it and describe its motion.

3. Plot the A and B data after removing the decay by dividing by the decaying exponential exp(-γ*t*/2).

 a. What is the "beat" frequency from the beat period?

 b. How would you curve fit the beats?

 c. Compute the value of the coupling capacitor Ccoupling from the beat frequency.

 d. Measure Ccoupling directly (without removing it from the box) and explain how you measured it.

(Figure out how to separate it from the circuit.)

4. Discuss what happens to the individual waveforms when the two uncoupled oscillators have slightly different frequencies, but are still coupled?

5. Discuss any differences in the waveforms for the rising and falling edges of the excitation pulse.



**IV. APPENDIX: SIMPLE NOTES ON DRIVEN DAMPED HARMONIC OSCILLATOR**

This page derives formulas for mechanical and electrical harmonic oscillators that have damping.

The next page derives formulas for a harmonic oscillator (HO) that is driven externally.

+++++++++++++++++++++++++++++++++++++++++ **Mechanical HO**



The damped *force* equation

for the *position* *x*(*t*) is



Here, *m* is the mass, β the velocity-dependent damping constant, and *k* the spring constant. Using the following substitutions

 **γ =** β**/*m*** and **ωo 2 = *k/m***.

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**LRC Electrical HO**

Here *L* is the inductance, *R* is the resistance, *C* is the capacitance, and *i*=d*q*/d*t* is the current. Using the following substitutions

 **γ= *R*/*L*** and **ωo 2 = 1/*LC***.

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The above equations can be rewritten generally as

Solutions are found by substituting x(t)=Re{Aea t} into the differential equation, then

[α2+ α γ+ ωo2] x(t) = 0, or [α2+ α γ+ ωo2] = 0, and α = ½ [–γ ± (γ2–4 ωo2)1/2].

The solution for the underdamped case, γ<< ωo, is a

sinusoid with a decaying amplitude given by

The damping constant γ has the same units of frequency as the oscillating frequency ωo. The angular frequency ω has units of radians/sec, or simply s–1. The frequency *f* has units of Hertz (cycles/sec), where ω=2π *f*. The period of oscillation is ***T* = 1/*f* = 2π/ω**.

Note that the damping reduces the frequency from ωo to ω’ = (ωo2 – γ2/4)1/2 = ωo (1 – γ2/4 ωo2)1/2.

The “quality factor” or Q-factor is a dimensionless quantity given by the ratio of frequency to damping,









