Advanced Physics Lab – Errors and Uncertainties

Advanced Physics Lab, PHYS 3600
Don Heiman, Northeastern University, 2021

This Week

1a-INTRO-a & 1b-INTRO-b: Introduction to the Course
motivation, boiler plate (syllabus/labs)
Fermi questions, exercises

2-ERRORS: Errors and Uncertainties
accuracy, precision, round off, propagation of errors

3-OPTICS: Optical Properties
EM spectrum, photo detectors, light emitters

4-SEMICOND: Semiconductors
band gap, Fermi energy, resistivity, Hall effect

5-ACOUSTICS:
sound, beats, Fourier transform, music

6-EXPERIMENTS: Intro to Lab Experiments
Virtual tour my research lab

Errors and Uncertainties

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Notes are on the website

1. **Precision** versus **Accuracy**
2. **Types of Errors**
3. **Significant Digits** and **Round-off**
4. Mean and Standard Deviation
5. **Propagation of Errors**
6. Interpolation
7. Smoothing y(x) Curves

Precision versus Accuracy

1.1 Precision →

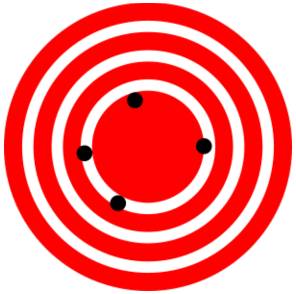
- How ***exact*** is a measurement, or how "*fine*" is the scale (# of significant figures) – **does not mean it is correct**

(also referred to as repeatability or reproducibility)

Suppose you measure a resistor with a digital ohmmeter. The ohmmeter reads 1.53483 Ω. This number has a high *precision*. However, it may not represent the "true" resistance as the wires connecting the resistor and ohmmeter have some small resistance that contributes to the measurement.

1.2 Accuracy →

– How ***close*** is a measurement to the "*true*" value



Accuracy is a measure of the ***correctness*** of the measurement. To determine a more *accurate* value for the resistor's resistance, you could measure the resistance of the wires and subtract it.

Precise and Accurate →

Types of Errors

We often want to know how close we are to the truth. The ***error*** is simply the **quantifiable difference** between the value obtained in real life and the "***true***" value.

There are three main categories of errors:

blunders, systematic errors, and random errors.

2.1 Blunders – These can usually be avoided by examining the results as you proceed with measurements.

Examples: reading and recording the wrong scale on the instrument, such as **millivolts instead of volts**; reading **milliseconds instead of microseconds** on the oscilloscope time base; forgetting to convert frequency "f " in cycles/second (Hz) to "ω" in radians/second; or mixing up **centimeters and inches**.

2.2 Systematic Errors – these are "**reproducible**" errors from faulty calibration or biased observation.

They can often be estimated from analysis of the experimental techniques. If you are measuring a distance with an **incorrect ruler** (shortened or lengthened), you can calibrate the ruler and make the correction by multiplying the original distances by a calibration factor.

2.3 Random Errors – these are due to fluctuations in measurements from repeated experiments.

Random errors are due to **statistical fluctuations**, often referred to as noise, or limited precision of an instrument. They can be minimized by repeating the experiment many times. Note that random errors which are small produce high precision, but not necessarily high accuracy. The precision usually increases only as the **square root** of the measurement time or number of measurements.

Uncertainty, Significant Digits and Round-off

Uncertainty is a quantitative measure of the **Margin of Error**

Uncertainties are **not precise** (not exact)

Standard deviations or uncertainties are rounded off to ***one*** significant digit (sometimes two digits if the most significant digit is between"1" and "1.4").

Example, L = 0.04573 ± **0.00253** m

(1) Round off the uncertainty first,
L = 0.04573 ± **0.003** m

(2) Then round off the leading number at the ***same digit*** as the uncertainty.
0.04**6** ± 0.00**3** m

or **(4.6 ± 0.3) x 10^{-2} m**

or **4.6 ± 0.3 cm (note leading zero)**

or simply **46 ± 3 mm**

Homework Exercises – Round off

Write the numerical values with the correct round offs.

1. $A = 0.012354 \pm 0.000354\ \Omega$

   A = ______________________ ± _____________ Ω

2. $B = 25.634\ m \pm 2.732\ m$

   B = ______________________ ± ____________________ m

3. $C = 789.3945\ s \pm 0.567\ s$

   C = __________________± ____________ s

Propagation of Errors

5.1 Addition and Subtraction

$$z(x,y) = ax \pm by$$

Given the function $z(x,y) = ax \pm by$, where x and y are not correlated and their standard deviations are σ_x and σ_y, the standard deviation in z is given by

$$\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2.$$

For example, compute the perimeter of a rectangle with sides H=2.0 ± 0.2 cm and W=3.5 ± 0.4 cm. The perimeter is P = H+H+W+W = 2H+2W=11.0 cm.
The standard deviation in the perimeter is

$$\sigma_P^2 = 2^2 * \sigma_x^2 + 2^2 * \sigma_y^2$$
$$\sigma_P^2 = 2^2 * 0.2^2 + 2^2 * 0.4^2 = 0.9^2.$$

Thus, the resulting perimeter is **P = 11.0 ± 0.9 cm**.

For simple addition of two numbers (**z=x+y**) the standard deviations are added in **quadrature**

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2.$$

Propagation of Errors

Trick #1

Eliminating negligible uncertainties in quadrature addition/subtraction

Suppose that you are given the values

$A = 100 \pm 1$

$B = 100 \pm 10$

$C = 100 \pm 30$

The sum $S = A+B+C = 300$

$$\sigma_S = [\, 1^2 + 10^2 + 30^2 \,]^{1/2}$$
$$= [\, 1 + 100 + 900 \,]^{1/2}$$
$$= 31.6$$

$= 30$ when rounded off

= the largest uncertainty

Rule – eliminate uncertainties that are at least 3-times smaller.

Propagation of Errors

5.2 Multiplication and Division

$z(x,y) = ax^n y$ or $z(x,y) = ax^n/y$

Given **$z(x,y) = ax^n y$** or **$z(x,y) = ax^n/y$**, where x and y are not correlated and their standard deviations, are σ_x and σ_y, then the standard deviation in z is determined by the ***fractional* uncertainties**, **σ_x/x** and **σ_y/y** .

$$(\sigma_z/z)^2 = n^2(\sigma_x/x)^2 + (\sigma_y/y)^2$$

For example, compute the area of the rectangle with sides H=2.0 ± 0.2 cm and W=3.5 ± 0.4 cm. The area is A=HW=7.0 cm^2. The standard deviation in the area is now

$$\sigma_A^2 = A^2 [(\sigma_x^2 / x^2) + \sigma_y^2 / y^2]$$
$$\sigma_A^2 = A^2 [(0.2^2/2.0^2) + 0.4^2/3.5^2] = 7^2 * 0.15^2.$$

Thus, the resulting area is **A=7 ± 1 cm^2**.

Propagation of Errors

Trick #2

Eliminating negligible uncertainties

in quadrature multiplication/division

Suppose that you are given the values

$A = 10 \pm 3$

$B = 100 \pm 10.$

Which has the largest fractional uncertainty?

$P = A*B$

$\sigma_P / P = [\ (3/10)^2 + (10/100)^2]^{1/2}$

$= [(0.3)^2 + (0.1)^2]^{1/2}$

$= 0.316$

$= 0.3 = 3/10$ when rounded off

= the largest fractional uncertainty

$P=A*B = 1000 \pm 1000*0.3 =$ **1000 ± 300**

P has 30 % uncertainty, same as uncertainty in A

Rule – eliminate **fractional** uncertainties that are at least 3-times smaller, even if the magnitude of the uncertainty is larger.

Propagation of Errors

Generalization

For any function $F(x,y,...)$,
you can compute the total uncertainty using

$$\sigma_F^2 = (\partial F/\partial x)^2\, \sigma_x^2 + (\partial F/\partial y)^2\, \sigma_y^2 + \ldots\ ,$$

where the x,y… variables are independent and not correlated.

5.3 Exponents

For $z = a\, x^{\pm b}$, $\sigma_z/z = b\, \sigma_x/x$

For $z = a\, e^{\pm bx}$, $\sigma_z/z = b\, \sigma_x$

For $z = a\, \ln(\pm bx)$, $\sigma_z = a\, \sigma_x/x$.

Summary - Propagation of Errors

Addition/Subtraction
Eliminating negligible uncertainties in quadrature

Suppose that you are given the values

$A = 100 \pm 1$

$B = 100 \pm 10$

$C = 100 \pm 30$

The sum is $S = A+B+C = 300$

$\sigma_S = [\ 1^2 + 10^2 + 30^2\ ]^{1/2}$
$= [\ 1 + 100 + 900\ ]^{1/2}$
$= 31.6$
$= 30$ when rounded off
= the largest uncertainty

Rule – eliminate **uncertainties** that are at least 3-times smaller.

Multiplication/Division
Eliminating negligible uncertainties in quadrature

Suppose that you are given the values

$A = 10 \pm 3$

$B = 100 \pm 10$

The product is $P = A * B = 1000$

$\sigma_P / P = [\ (3/10)^2 + (10/100)^2]^{1/2}$
$= [(0.3)^2 + (0.1)^2]^{1/2}$
$= [0.09 + 0.01]^{1/2}$
$= 0.316$
$= 3/10$ when rounded off
= the largest fractional uncertainty

Rule – eliminate **fractional uncertainties** that are at least 3-times smaller, even if the magnitude of the uncertainty is larger.

Propagation of Errors

Homework Exercises – Propagation of Errors

1. Compute the combined uncertainties. Use formulas and show all work.

   (a) $x = 1.2 \pm 0.1$ kg, $y = 48.76 \pm 0.12$ kg

   $z = x + y$ Answer: z = ___________________ ± ___________ kg

   (b) $d = 600 \pm 30$ cm, $t = 0.118 \pm 0.006$ s

   $v = d / t$ Answer: v = ___________________ ± ___________ cm/s

2. A standard container drum has a diameter $D=20.6\pm0.2$ in and height $H=38\pm1$ in.

   (a) What is its volume and uncertainty.

   Answer: V = ___________ ± ___________ in^3

   (b) Express the answer in gallons (1 gal $\equiv$ 231.0 in^3).

   Answer: V = ___________ ± ________ gal

END